

SATS and the new GCSE

Where are we going and how do we get there?

Debbie Morgan

National Centre
for Excellence in the
Teaching of Mathematics



KS2 Arithmetic Paper

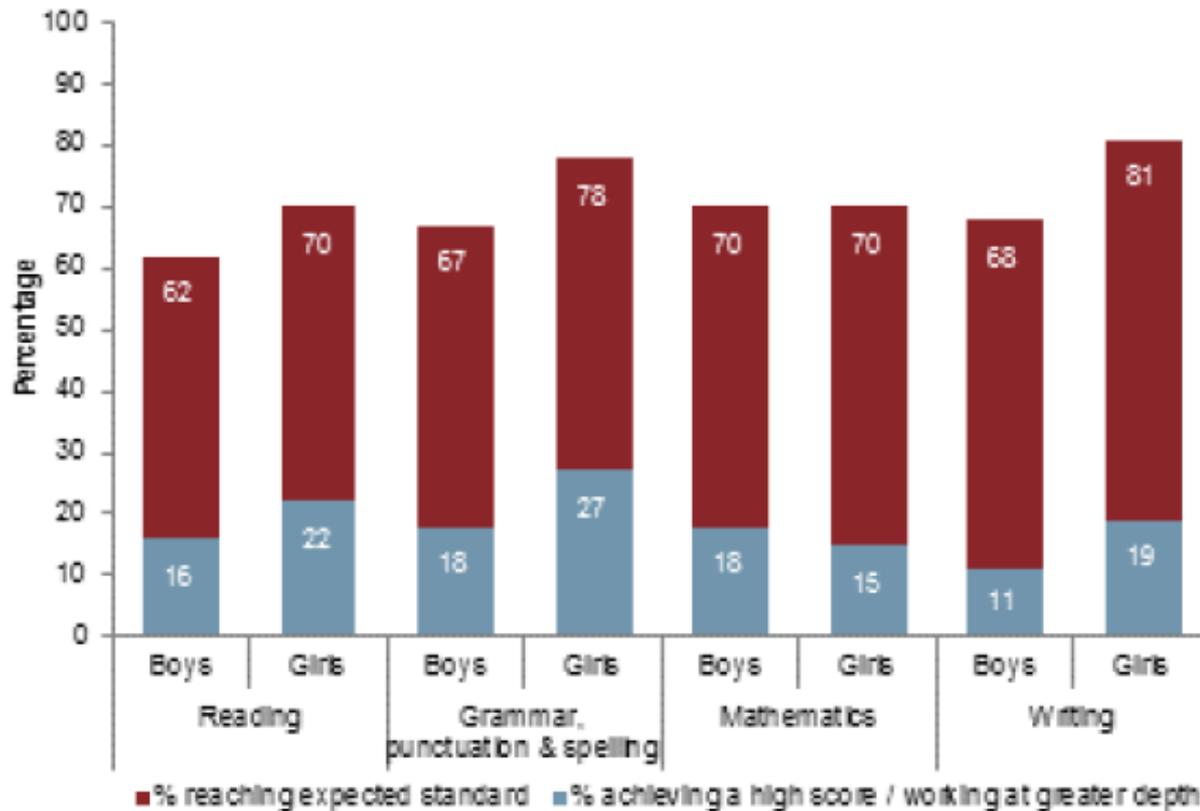
Debbie Morgan

National Centre
for Excellence in the
Teaching of Mathematics



2016

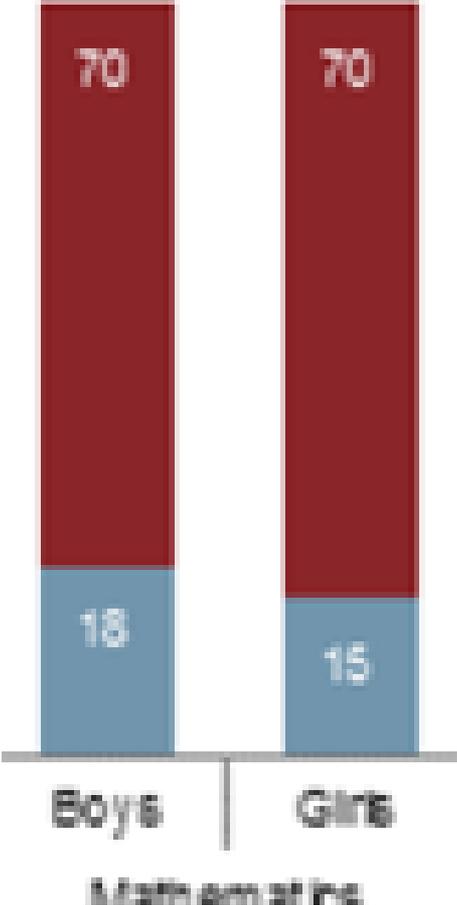
**Figure 5: Attainment by subject and gender
England, 2016 (all schools)**



Source: Provisional 2016 KS2 assessment data



2016 Gender Comparison



Scaled Scores

58	99
59	99
60	100
61	100
62	100
63	100
64	100
65	101
66	101
67	101
68	101

101	111
102	112
103	113
104	113
105	114
106	115
107	116
108	117
109	119
110	120

4	468 - 9 =					

Number

Correct: 94

A photograph of a student's handwritten work on grid paper. The student has written the subtraction $468 - 9 =$. The numbers are written in a grid. The top row contains '4', '6', and '8'. The second row contains a minus sign and '9'. A horizontal line is drawn under the '9'. The bottom row contains '4', '5', and '9'. The '6' in the top row has a small '5' written above it, and the '6' has a diagonal slash through it, indicating a borrowing error where the student subtracted 9 from 68 instead of 468.

KS2 How do you think children answered these?



4	$468 - 9 =$																																	
8	<div style="border: 2px solid blue; width: 150px; height: 40px; display: inline-block;"></div> $= 435 - 30$																																	
12	$50 \times 70 =$	<table border="1"><tr><td>16</td><td>$15.98 + 26.314 =$</td><td><table border="1"><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table></td></tr><tr><td>15</td><td>$486 \div 3 =$</td><td><table border="1"><tr><td>18</td><td>$122,456 - 11,999 =$</td><td><table border="1"><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table></td></tr><tr><td>20</td><td>$0.9 \div 10 =$</td><td></td></tr></table></td></tr></table>	16	$15.98 + 26.314 =$	<table border="1"><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>											15	$486 \div 3 =$	<table border="1"><tr><td>18</td><td>$122,456 - 11,999 =$</td><td><table border="1"><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table></td></tr><tr><td>20</td><td>$0.9 \div 10 =$</td><td></td></tr></table>	18	$122,456 - 11,999 =$	<table border="1"><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>											20	$0.9 \div 10 =$	
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$$326 \div 1 =$$

A photograph of a handwritten long division problem on grid paper. The problem is $1 \overline{)326}$. The quotient 326 is written above the dividend. Below the division line, the numbers 3 , 2 , and 6 are written in the respective columns. Below the grid, the text "Ans = 3 2 6" is written.

Number

Correct: 26

33

$$\frac{3}{5} \div 3 =$$

What mathematical structure of division do children need to think about to do this calculation easily?

3

Write the three missing digits to make this **addition** correct.

$$\begin{array}{r} 15\boxed{} \\ + 4\boxed{}4 \\ \hline \boxed{}15 \end{array}$$

Thinking about relationships

21

$$5,542 \div 17 = 326$$

Explain how you can use this fact to find the answer to 18×326

$$17 \times 326 = 5,542$$

$$18 \times 326 = 5,542 + 326$$

26%
correct

- How might children respond to this question?
- What is the best response?

Implications for Teaching – teaching with variation



$2 \times 3 =$	$6 \times 7 =$	$9 \times 8 =$
$2 \times 30 =$	$6 \times 70 =$	$9 \times 80 =$
$2 \times 300 =$	$6 \times 700 =$	$9 \times 800 =$
$20 \times 3 =$	$60 \times 7 =$	$90 \times 8 =$
$200 \times 3 =$	$600 \times 7 =$	$900 \times 8 =$

Practice exercises should draw attention to relationships

< > =

$$3.08 \div 0.5 \bigcirc 3.08$$

$$0.83 \div 1 \bigcirc 0.83$$

$$4.12 \div 1.2 \bigcirc 4.12$$

$$0.46 \div 0.21 \bigcirc 0.46$$

What would you want pupils to do?

What would they do?

Attention to 1 and 0

2	●	4	1	●	1	6	●	5	0	●	5	5	●	8
4	●	0	3	●	1	8	●	9	3	●	4	6	●	0
1	●	2	0	●	4	1	●	0	4	●	1	4	●	5
0	●	0	3	●	2	7	●	7	4	●	2	9	●	9

Examples from Shanghai textbooks

*In designing [these] exercises, the teacher is advised to avoid mechanical repetition and to create an appropriate path for **practising the thinking process** with increasing creativity.*

Gu, 1991

KS1 Assessment

Debbie Morgan

National Centre
for Excellence in the
Teaching of Mathematics



1

$$8 + 6 =$$

8

$$50 +$$

$$= 80$$

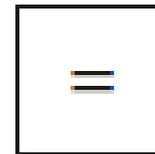
9

$$56 -$$

$$= 51$$

- Draw a part part whole model to answer these questions

29 Here are some signs.



Write the correct sign in each box.

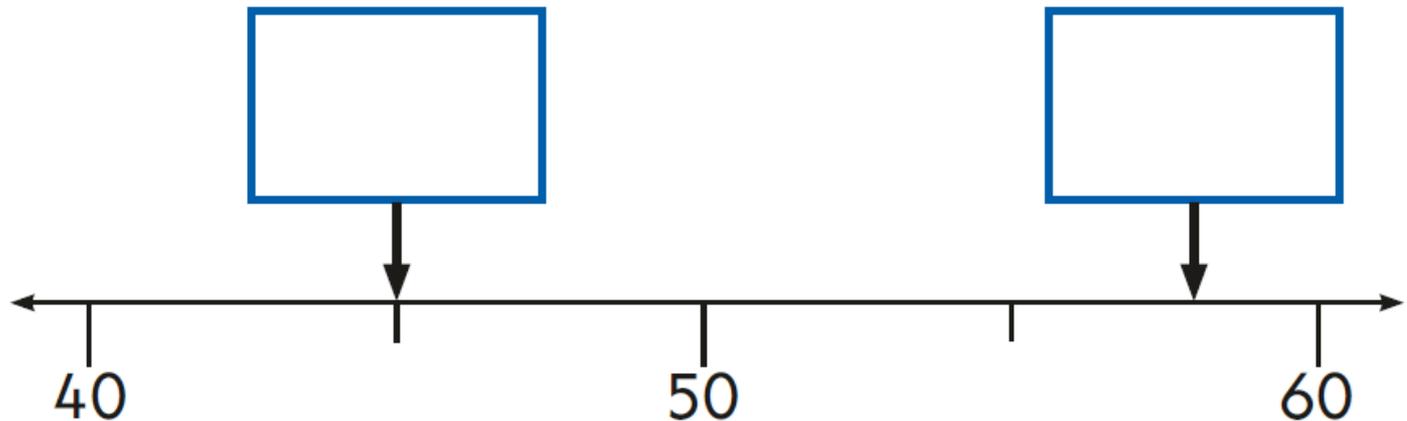
$$10 + 5 \quad \square \quad 10 \times 5$$

$$2 \times 6 \quad \square \quad 6 + 6$$

Thinking about numbers and the relationship between them

16

Write the correct number in each box.



How do children solve these- by calculating or looking at the relationships?



25

Match the sums that have the same answer.

One is done for you.

$$20 + 8$$

$$30 + 28$$

$$50 + 8$$

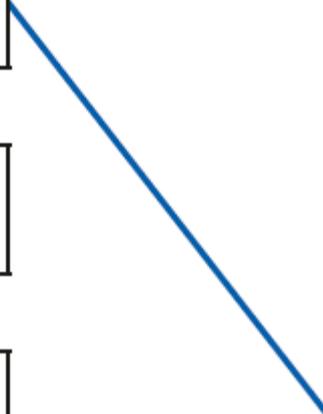
$$70 + 18$$

$$80 + 8$$

$$10 + 18$$

$$90 + 8$$

$$50 + 48$$



Looking at the same concept in different ways

An aspect of variation



2.	$10 + 1 = \square$	$10 + 4 = \square$	$10 + 9 = \square$	$10 + 7 = \square$
	$10 + 2 = \square$	$10 + 8 = \square$	$10 + 6 = \square$	$10 + 3 = \square$
3.	$14 - 4 = \square$	$17 - 7 = \square$	$16 - 6 = \square$	$19 - 9 = \square$
	$12 - 2 = \square$	$15 - 5 = \square$	$11 - 1 = \square$	$13 - 3 = \square$
4.	$17 = 10 + \square$	$13 = 10 + \square$	$18 = 10 + \square$	$12 = 10 + \square$
	$14 = 10 + \square$	$19 = 10 + \square$	$11 = 10 + \square$	$20 = 10 + \square$
5.	$10 + \square = 15$	$10 + \square = 14$	$10 + \square = 20$	$10 + \square = 17$
	$10 + \square = 12$	$10 + \square = 10$	$10 + \square = 16$	$10 + \square = 11$

*In designing [these] exercises, the teacher is advised to avoid mechanical repetition and to create an appropriate path for **practising the thinking process** with increasing creativity.*

Gu, 1991

QUIZ TIME

Primary (SATs)

Vs

Secondary (GCSE)

Which is Which?

Thankyou to Danielle Bartram

Maths Lead Practitioner

Acklam Grange School

Primary VS Secondary

$$70 \div \boxed{} = 3.5$$

Work out the missing number.

A

Use all the digits

0 1 5 0 1 5 0

to complete this multiplication:

$$\boxed{}\boxed{}\boxed{} \times 2 = \boxed{}\boxed{}\boxed{}\boxed{}$$

B

Primary VS Secondary

$$70 \div \boxed{} = 3.5$$

Work out the missing number.

Use all the digits

0 1 5 0 1 5 0

to complete this multiplication:

$$\boxed{}\boxed{}\boxed{} \times 2 = \boxed{}\boxed{}\boxed{}\boxed{}$$

Primary

Secondary

Primary VS Secondary

Lara chooses a number less than 20

She divides it by 2 and then adds 6

She then divides this result by 3

Her answer is 4.5

What is the number she started with?

A

Here is a number machine.



(a) Work out the **output** when the input is 4

(b) Work out the **input** when the output is 11

B

Primary VS Secondary

Lara chooses a number less than 20

She divides it by 2 and then adds 6

She then divides this result by 3

Her answer is 4.5

What is the number she started with?

Here is a number machine.



(a) Work out the **output** when the input is 4

(b) Work out the **input** when the output is 11

Primary

Secondary

Primary VS Secondary

The total weight of 3 tins of beans and 4 jars of jam is 2080 g.
The total weight of 5 tins of beans is 2000 g.

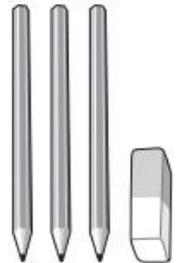
Work out the weight of 1 tin of beans and the weight of 1 jar of jam.

A

6 pencils cost **£1.68**



3 pencils and 1 rubber cost **£1.09**



What is the cost of a rubber?

B

Primary VS Secondary

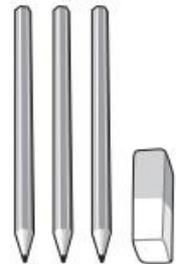
The total weight of 3 tins of beans and 4 jars of jam is 2080 g.
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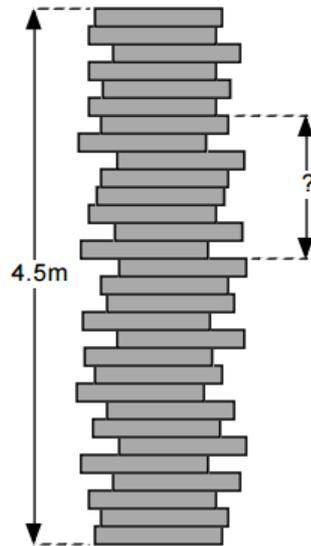
Secondary

Primary

Primary VS Secondary

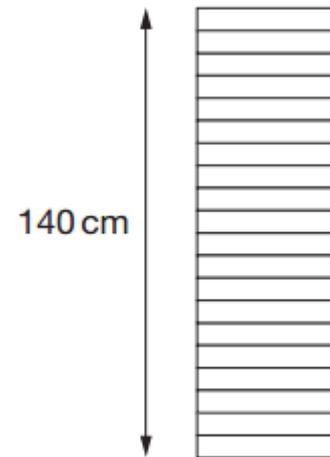
A tower of 30 identical wooden blocks is 4.5 metres tall.

What is the distance from the top of the 16th block to the top of the 24th block?



A

A stack of 20 identical boxes is 140 cm tall.



Not
actual
size

Stefan takes **three** boxes off the top.

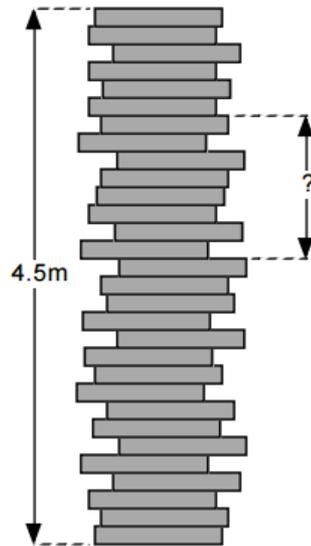
How tall is the stack now?

B

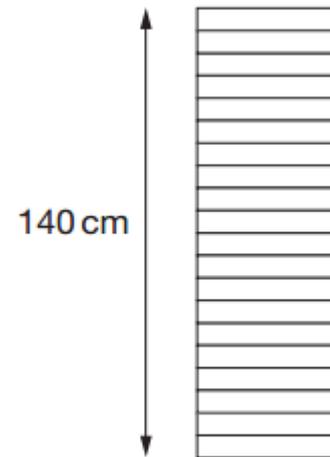
Primary VS Secondary

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**Not
actual
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Stefan takes **three** boxes off the top.

How tall is the stack now?

Secondary

Primary

Primary VS Secondary

Miss Mills is making jam to sell at the school fair.

Strawberries cost £7.50 per kg.

Sugar costs 79p per kg.

10 glass jars cost £6.90

She uses 12 kg of strawberries and 10 kg of sugar to make 20 jars full of jam.

Calculate the total cost to make 20 full jars?

A

Faiza buys

one magazine costing £2.30

one paper costing 92p

two identical bars of chocolate

Faiza pays with a £5 note.

She gets 40p change.

Work out the cost of **one** bar of chocolate.

B

Primary VS Secondary

Miss Mills is making jam to sell at the school fair.

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Primary

Secondary

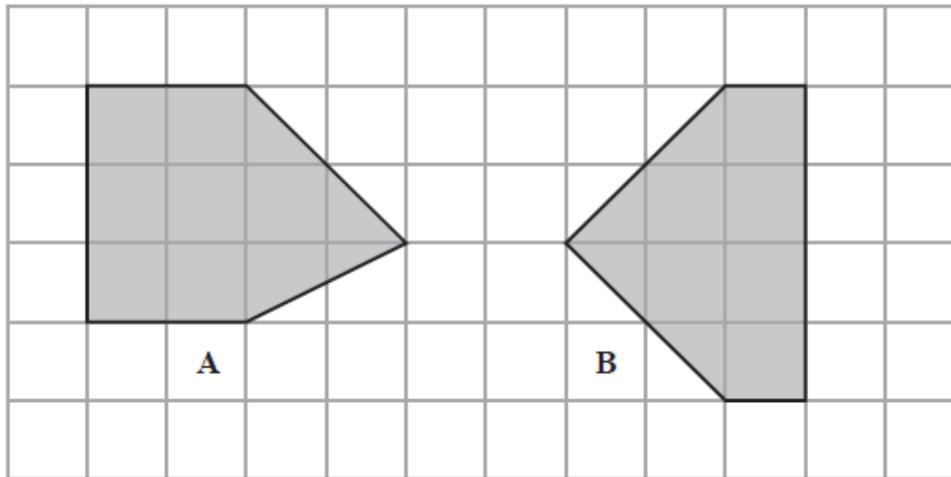
Reasoning Methods and GCSE

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How do we view it?

*9 Shape A and shape B are drawn on a centimetre grid.



One shape has a greater area than the other.
How much greater?

What are their first thoughts:

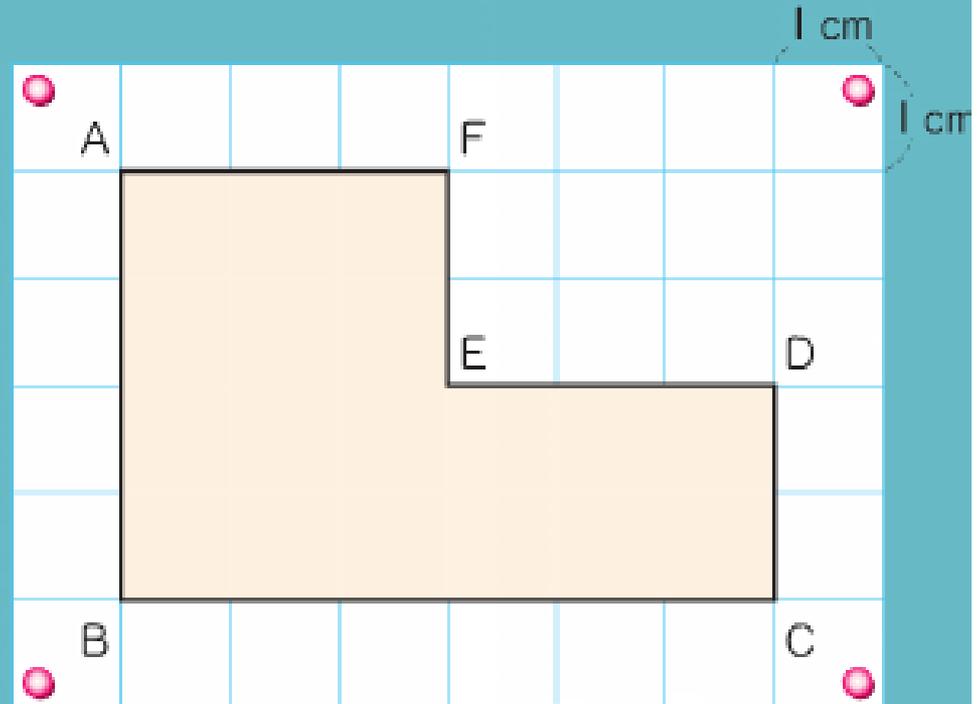
Can I remember the formulae for a shape like this?

Lets count the squares

Implications for Teaching

Methods for finding area

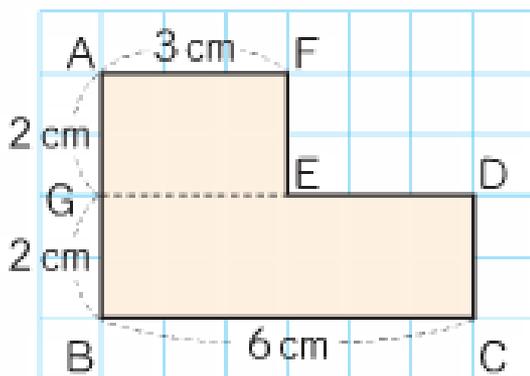
2 Find the area of the shape on the right.



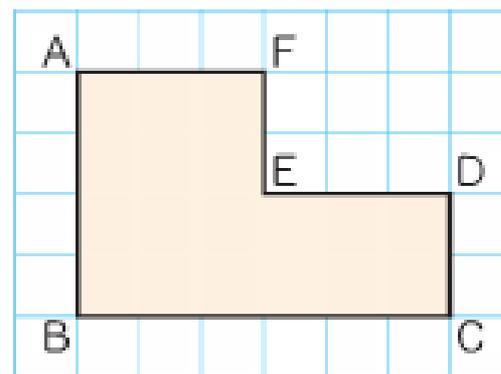
Analysis of methods to think more deeply and develop efficient strategies

Miho and her classmates are explaining the their friends' ideas.

Hiroki

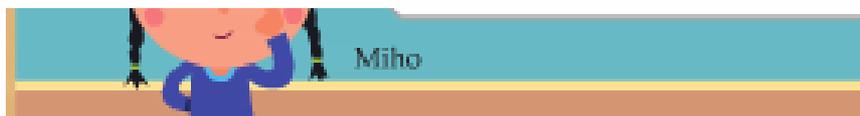


Takumi



$$4 \times 6 - 2 \times 3 = 24 - 6 \\ = 18$$

2 Look at what Hiroki drew and write down his ideas using math sentences.



• Providing Textbook Supports for Teaching Math Akihiko Takahashi
<https://prezi.com/s1nvam1qllv9/providing-textbook-supports-for-t>

3 Look at the math sentence Takumi wrote and explain how he thought about the problem.



Write down the lengths of the segments and draw in any additional segments in the figure above.

Developing Flexibility of Thinking

The less successful seem to focus more on perceptions of their physical activities than on the flexible use of symbol as process and concept appropriate for a conceptual development in arithmetic and algebra

Repetition leading to automation

Although such repetition and exteriorisation of procedures has been seen as an essential part of mathematics learning, for decades it has been known that it has made no improvement to **understanding of relationships**. More importantly, if used exclusively, it may lead to a form of procedural thinking that lacks the flexibility necessary to solve novel problems.

How are students tackling this problem?

- 4 The table shows the numbers of people, in thousands, visiting five castles in 2009 and in 2010

Castle	Number of people (thousands)	
	2009	2010
Bodiam	185	177
Chirk	108	120
Dunster	133	128
Penrhyn	231	230
Scotney	124	118

-8
+12
-5
-1
-6

Edexcel
Foundation
Paper A

- *(a) Did a greater total number of people visit these castles in 2009 or in 2010?
You must show all your working.



10 Here is a list of numbers.

4 6 8 11 15 33 44

(a) From the list, write down a factor of 42

(b) From the list, write down a multiple of 22

Part 3 is mastery –
aiming for all students
to answer this
question – that’s
where the reasoning
lies

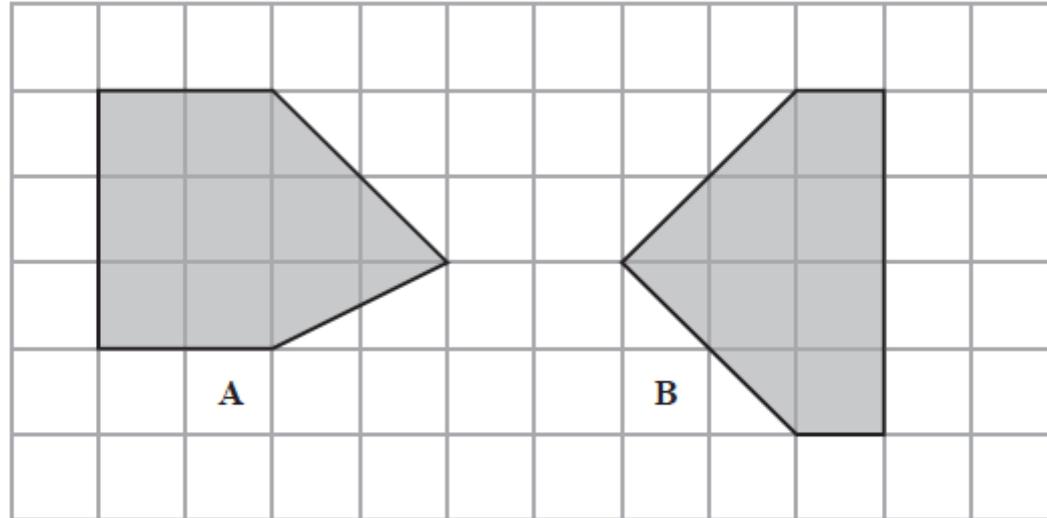
(c) From the list, write a different number in each box to make the statement true.

Edexcel
Foundation
Paper A

$$\square \div \square \times \square = 12$$

How do we view it?

*9 Shape A and shape B are drawn on a centimetre grid.



Edexcel
Foundation
Paper A

One shape has a greater area than the other.
How much greater?

Edexcel
Foundation
Paper A

2017 Reasoning and Relationship between numbers



Here are some numbers.

9.6

12.6

15.4

7.6

12.4

17.4

Write the numbers in pairs so that the **sum** of the numbers in each pair is the same.

[2 marks]

9.6 12.6
7.6 15.4
12.4 17.4

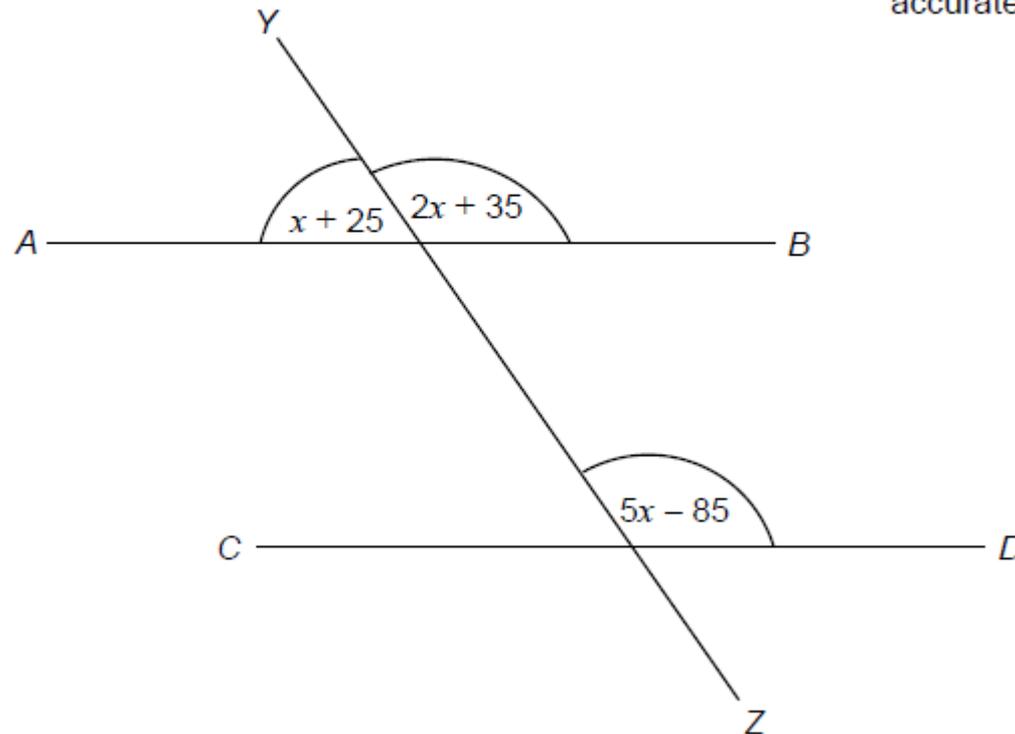
AQA Sample Paper

2017

AB , CD and YZ are straight lines.

All angles are in degrees.

Not drawn
accurately



Show that AB is parallel to CD .

Avoiding Overload

Children need to know key facts and ensure these are maintained:

Additive Facts

Times Tables

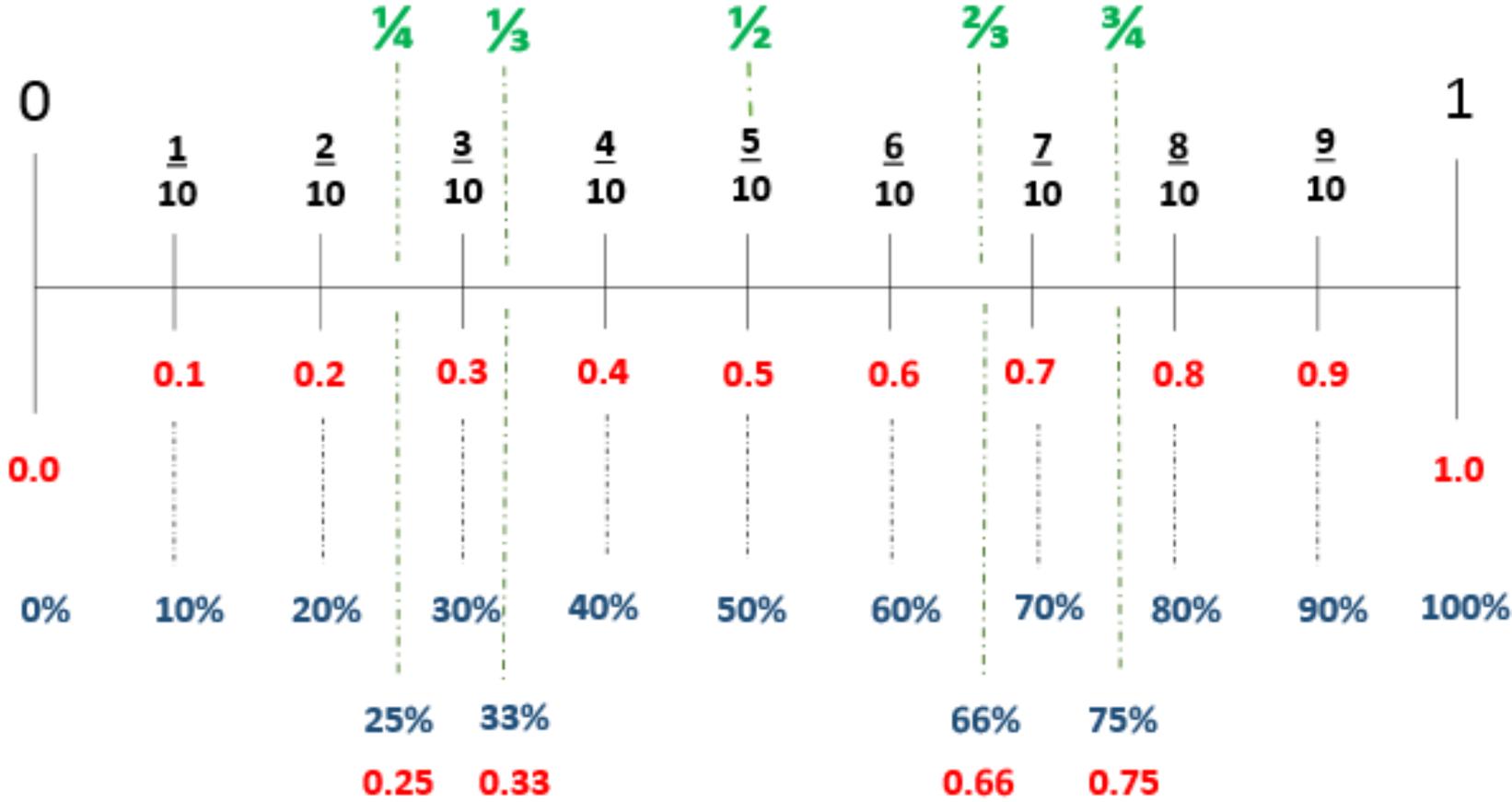
Key fraction, decimal and percentage

Equivalents

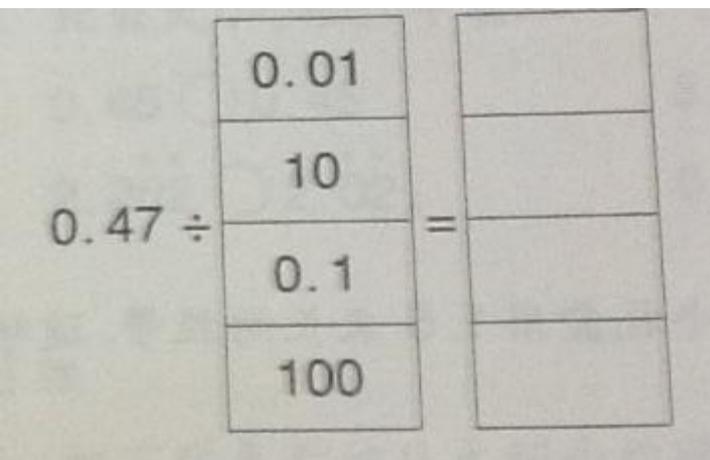


Key Facts

Equivalent Fractions, Decimals & Percentages



Reasoning tasks to make connections and develop fluency



3. 根据“ $45 \times 98 = 4410$ ”, 在下列算式的得数中点上小数点, 使算式成立:

(1) $4.5 \times 9.8 = 4410$

(2) $4.5 \times 0.98 = 4410$

(3) $0.45 \times 0.098 = 4410$

(4) $0.045 \times 9.8 = 4410$

1. 比较大小, 在○中填入“>”、“<”或“=”:

$3.35 \times 0.9 \bigcirc 3.35$

$12.85 \times 0.5 \bigcirc 12.85$

$9.6 \times 0.2 \bigcirc 0.2$

$0.76 \times 1 \bigcirc 0.76$

$45.3 \times 2.1 \bigcirc 45.3$

$16.3 \bigcirc 1.8 \times 16.3$

$1.39 \times 2.7 \bigcirc 1.39$

$2.78 \times 3.4 \bigcirc 2.78$

$7.4 \bigcirc 7.4 \times 0.7$

$3.23 \times 1.9 \bigcirc 3.23$

$0.37 \times 0.6 \bigcirc 0.37$

$4.8 \bigcirc 4.8 \times 12.1$

$$7 + 2 = \quad 9 + 6 =$$

$$17 + 2 = \quad 10 + 6 =$$

$$7 + 12 = \quad 11 + 6 =$$

$$17 + 12 = \quad 13 + 6 =$$

$$9 - 5 = \quad 9 - 7 =$$

$$8 - 5 = \quad 11 - 7 =$$

$$7 - 5 = \quad 13 - 7 =$$

$$6 - 5 = \quad 15 - 7 =$$

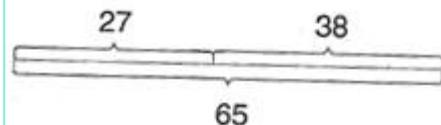
$$18 - \square = 8 \quad 20 - \square = 16$$

$$18 - \square = 10 \quad 18 - \square = 10$$

$$18 - \square = 12 \quad 16 - \square = 12$$

$$18 - \square = 14 \quad 14 - \square = 6$$

$$18 - \square = 16 \quad 12 - \square = 8$$

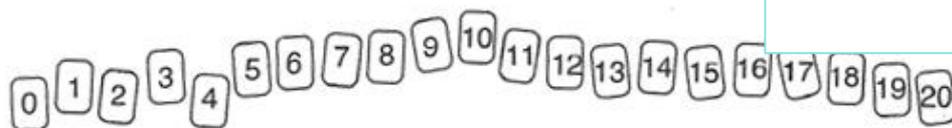


$$\square + \square = \square$$

$$\square + \square = \square$$

$$\square - \square = \square$$

$$\square - \square = \square$$



$$9 \times 50 \bigcirc 90 \times 5$$

$$300 \times 3 \bigcirc 5 \times 200$$

$$14 - \dots < 6$$

$$907 - 100 = 807$$

$$907 - 99 = \underline{\quad}$$

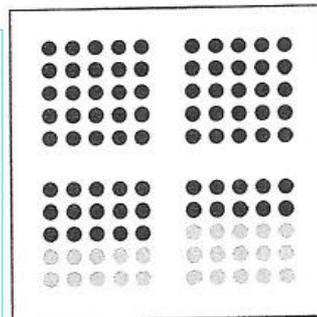
$$907 - 101 = \underline{\quad}$$

推

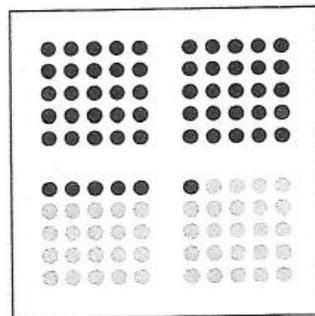
$$888 - 99 = \underline{\quad}$$

$$888 - 100 = \underline{\quad}$$

$$888 - 101 = \underline{\quad}$$



$$75 + \dots = 100$$



$$56 + \dots = 100$$

Variation: Applying the same operation to connected numbers

A級

1. 推算。

(1)

4	$\times 12 =$	
40		
400		

(2)

$43 \times$	2	$=$	
	20		
	200		

(3)

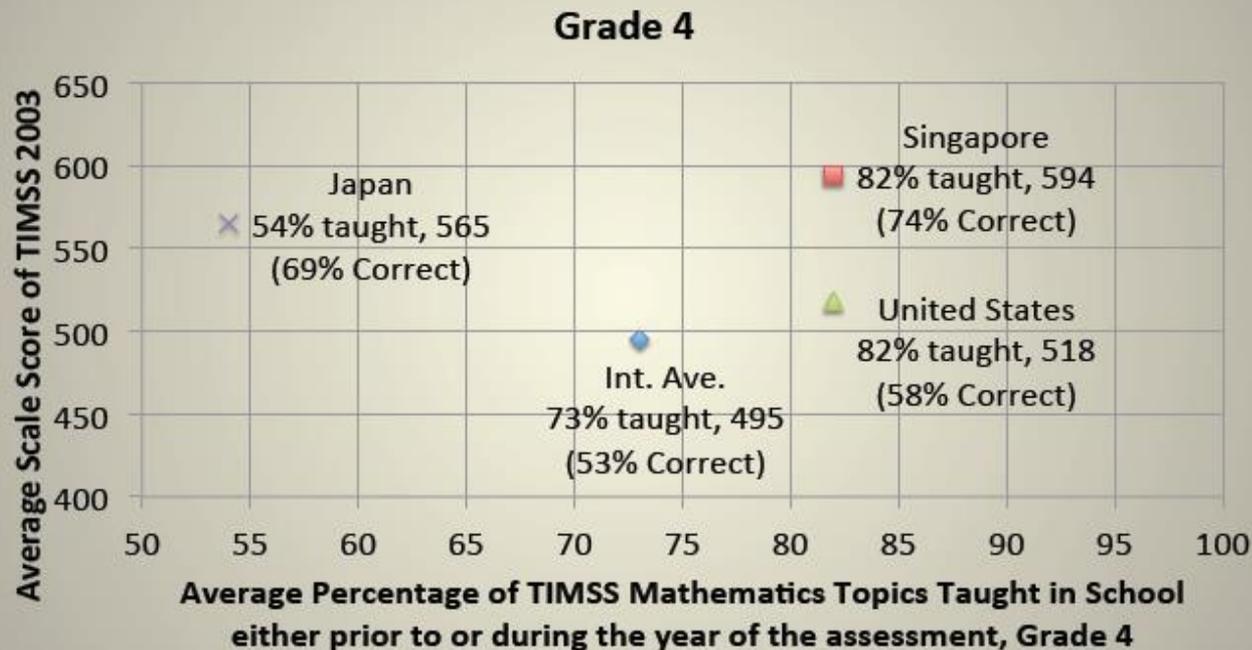
5	$\times 16 =$	
500		
50		

(4)

$27 \times$	30	$=$	
	3		
	300		

The relationship between coverage and test results

Average Percentage of TIMSS Mathematics Topics Taught in School and the Achievement (Average Scale Score) of the TIMSS 2003

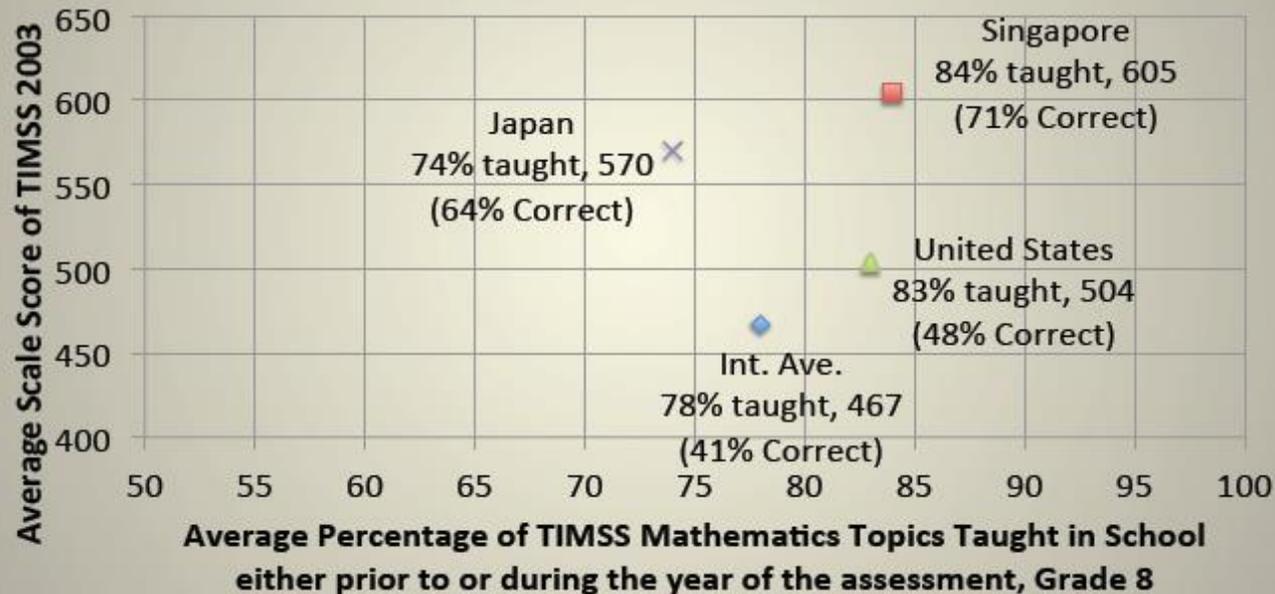


Source TIMSS 2003 International Mathematics Report
Grade 8: Exhibit 5.7 (p.192), Exhibit C. 1 (p.400)
Grade 4: Exhibit 5.7 (p.193), Exhibit C. 1 (p.402)

Thankyou to Dr Akihiko Takahashi for this slide

Average Percentage of TIMSS Mathematics Topics Taught in School
and the Achievement (Average Scale Score) of the TIMSS 2003

Grade 8



Source TIMSS 2003 International Mathematics Report
Grade 8: Exhibit 5.7 (p.192), Exhibit C. 1 (p.400)
Grade 4: Exhibit 5.7 (p.193), Exhibit C. 1 (p.402)

LESSON STUDY ALLIANCE

Helping teachers work together to improve teaching & learning.

<http://www.LSAlliance.org>

Thankyou to Dr Akihiko Takahashi for this slide

Some pupils are tempted to evade precisely that portion of the work which gives the benefit, by memorizing the results of the work of others. This temptation is great to some pupils, and perhaps no other subject can become so barren and dreary as mathematics so studied. **Memorizing.** Ten pages of mathematics *understood* are better than a hundred memorized and not understood, and one page actually worked out independently is better than ten pages clearly but passively understood. The question is not *how much?* but *how?* The object is mastery, attainment of the spirit of the subject, and not to train the memory, or to ingest a large bulk of mathematical fact and formulas.]

(J.W.A. Young, 1908, p.38)

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Learning mathematics and doing well in tests

- Avoiding overload – knowing key facts
- Variation both in teaching and practice
- Slow down
- Go Deeper
- Focus on relationships
- Avoid mechanical repetition, but practice the thinking process
- Take a risk and cover less

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