Problem Solving and Reasoning

session 3

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Aims of the 3 sessions

- Session 1 – What is NRICH?

  NRICH as a tool for planning and teaching. Problem solving and reasoning in the National Curriculum.

  Gap Task.
- Session 2 - Reflection on the Gap Task.

  NRICH planning looking at your medium term plan, adapting to incorporate fluency and reasoning.

  Second Gap Task.
- Session 3 - Reflection on the Second Gap Task.

  Understanding the different types of problem solving.

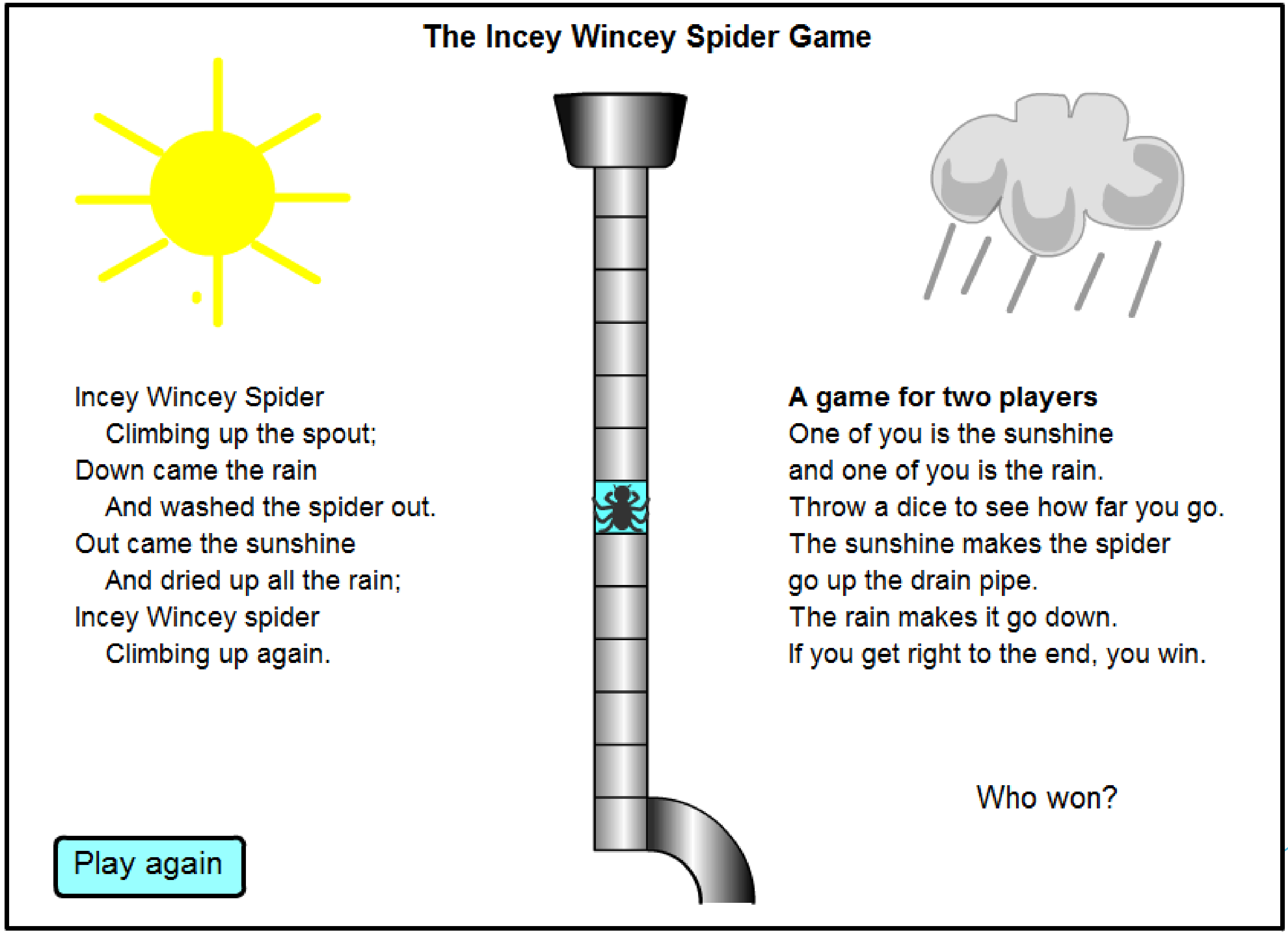
  Ways forward.

Aims of the Session

- To reflect on the Gap Task and share thoughts.
- To look at different types of reasoning using NRICH.
- To think about how we can move forward following these workgroups.

Incey Wincey Spider

- https://nrich.maths.org/8863



Gap Task

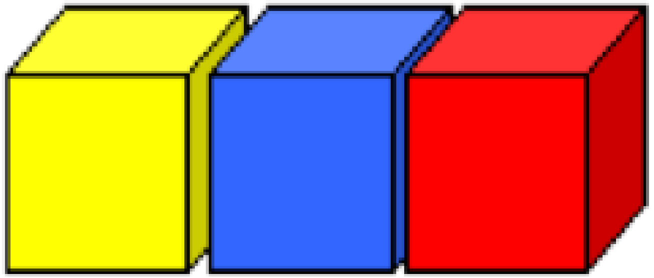
- How did you get on?
  - What year group?
  - Which activity? How did you adapt it?
  - Did if go well?
  - Did anything not go well?
  - How would you change it to use it again?
  - Children’s views.

Add 3 Dice

- https://nrich.maths.org/1016

Place three dice in a row.

Find a way to turn each one so that the three numbers on top of the dice total the same as the three numbers on the front of the dice.



Can you find all the ways to do this?

(Look at the totals on the back and bottom of the three dice – what do you notice?)

Reasoning

- The national curriculum for mathematics aims to ensure that all pupils:
  - become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
  - **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
  - can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

What is the difference between Problem Solving and Reasoning?

Consider the following problem: If each bus holds 44 students, how many buses are needed so that all 154 third graders can go to the zoo?

- Students need to evaluate the problem then represent it symbolically. When they complete the calculation, they will arrive at the answer 3 ½. Often students stop here. After all, they have an answer so they must be done. What’s missing is the reasoning part of solving the problem. If students reasoned, they would realize that it is impossible to have 3 ½ buses. 4 buses are needed in order to transport all the students to the zoo.

Reasoning: Identifying Opportunities

- https://nrich.maths.org/10990
- Reasoning is fundamental to knowing and doing mathematics. We wonder how you would define the term? Some would call it systematic thinking. Reasoning enables children to make use of all their other mathematical skills and so reasoning could be thought of as the 'glue' which helps mathematics makes sense.

  The second aim of the new mathematics national curriculum in England (DfE, 2013) is that all pupils will:

  reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.
- In order to explore this aim, three questions need to be answered: When is reasoning necessary?
- What do we do when we reason?
- How do we support children to develop their reasoning skills?

Reasoning is needed, when …

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- … first encountering a new challenge
- … logical thinking is required
- … a range of starting points is possible
- … there are different strategies to solve a problem
- … there is missing information
- … selecting a problem-solving skill
- … evaluating a solution in context
- … there is more than one solution

Five-step progression in reasoning

- Step 1: **Describing:** simply tells what they did.

  Step 2: **Explaining:** offers some reasons for what they did. These may or may not be correct. The argument may yet not hang together coherently. This is the beginning of inductive reasoning.

  Step 3: **Convincing:** confident that their chain of reasoning is right and may use words such as, 'I reckon' or 'without doubt'. The underlying mathematical argument may or may not be accurate yet is likely to have more coherence and completeness than the explaining stage. This is called inductive reasoning.

  Step 4: **Justifying:** a correct logical argument that has a complete chain of reasoning to it and uses words such as 'because', 'therefore', 'and so', 'that leads to' ...

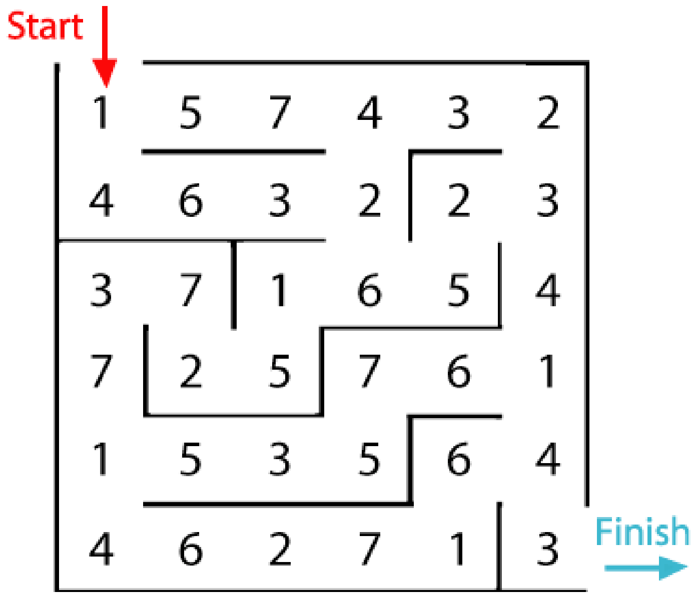
  Step 5: **Proving:** a watertight argument that is mathematically sound, often based on generalisations and underlying structure. This is also called deductive reasoning.

Maze 100 Is this Reasoning?

- https://nrich.maths.org/91

In this maze there are numbers in each of the cells. You go through adding all the numbers that you pass. You may not go through any cell more than once.

Can you find a way through in which the numbers add to exactly 100?



What is the lowest number you can make going through the maze?

What is the highest number you can make going through the maze?
(Remember you may not go through any cell more than once.)

Communicating reasoning

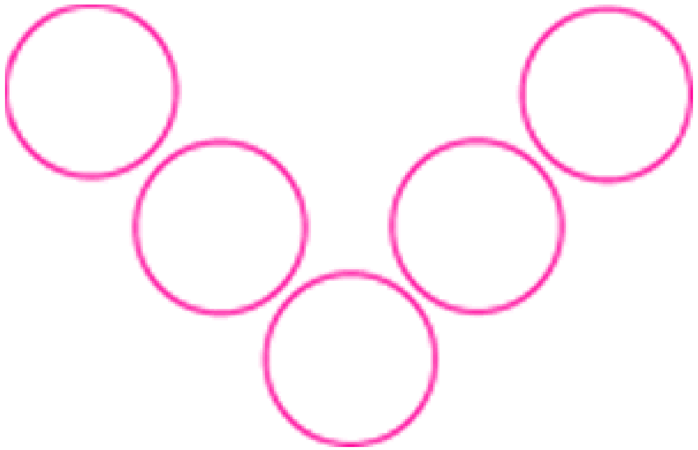
Here are some possible sentence starters:

- I think this because …
- If this is true then …
- I know that the next one is … because …
- This can’t work because …
- When I tried _________ I noticed that …
- The pattern looks like …
- All the numbers begin with …
- Because ________ then I think ________
- This won’t work because …

Magic V's

- https://nrich.maths.org/6274

Place each of the numbers 1 to 5 in the V shape below so that the two arms of the V have the same total.



How many different possibilities are there?
What do you notice about all the solutions you find?

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Maths Talk

<u>Trying out the 9 questions:</u>

1. Why is that a good mistake?
2. If we know this, what else do we know?
3. Give me . . .tell me . . .show me . . .?
4. Why is this the odd one out?
5. The answer is . . .what is the question?
6. Can you zone in?
7. Give me a silly answer for . . .?
8. Always, sometimes, never true?
9. Give me a **P**eculiar,**O**bvious,**G**eneral example.

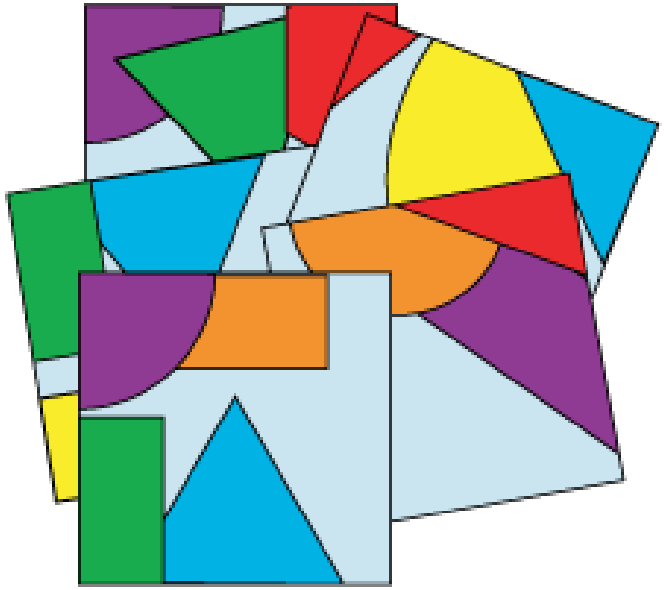
Talk Maths
Camden Maths Learning Network

Problem Solving:

In order to solve a problem, we need to draw on one or more problem-solving skills, such as:

- Working systematically
- Trial and improvement
- Logical reasoning
- Spotting patterns
- Visualising
- Working backwards
- Conjecturing

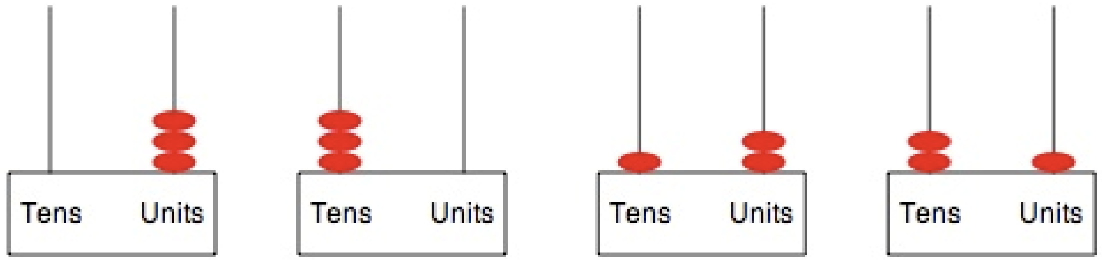
Jig Shapes



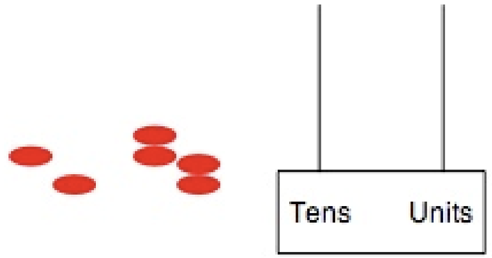
6 Beads

- https://nrich.maths.org/152 (Year 2)

If you put three beads onto a tens/units abacus you could make the numbers 3, 30, 12 or 21.

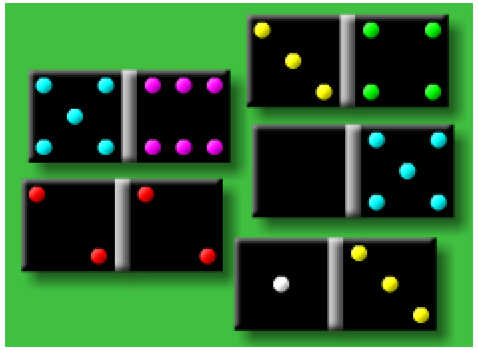


Explore the numbers you can make using six beads.



Domino Sets

- Dominoes are normally in a set of 0-6
- Before you start playing it might be a good idea to find out if you have a full set!



- How would you go about it?
- How could you be sure?

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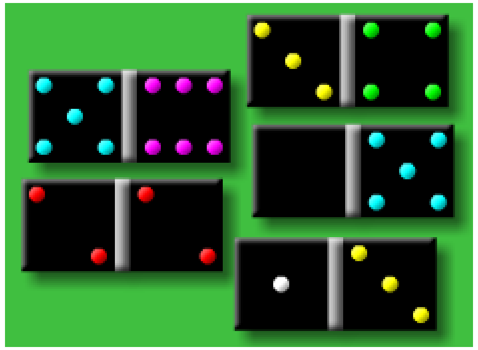

- How would you go about it?
- How could you be sure?

- What if someone gave you some 0-9 dominoes?
- How many do you think there would be in a full set?

Mathematical Powers – Mason(2004)

(The notion that learners make use of their natural powers to make sense of the world.)

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Specialising and Generalising

Specialising means to focus on one particular area in maths.

Generalising means to form an idea or conclusion about something.

Imagining and Expressing

Imagining means to form a mental image of something.

Expressing means communicate our idea to someone.

Organising and Classifying

Organising means to arrange your ideas and findings mathematically.

Classifying means to arrange your ideas into a particular group.

Conjecturing and Convincing

Conjecturing means to predict what might happen.

Convincing means to tell to someone what you have found out and explain why you're correct.

Biscuit Decorations

- https://nrich.maths.org/154 (Year 1)

Andrew decorated 20 biscuits to take to a party.

He lined them up and put icing on every second biscuit.

Then he put a cherry on every third biscuit.

Then he put a chocolate button on every fourth biscuit.

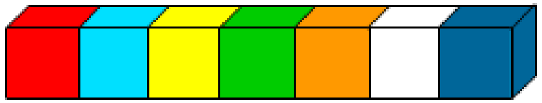
So there was nothing on the first biscuit.

How many other biscuits had no decoration? Did any biscuits get all three decorations?

Break it Up

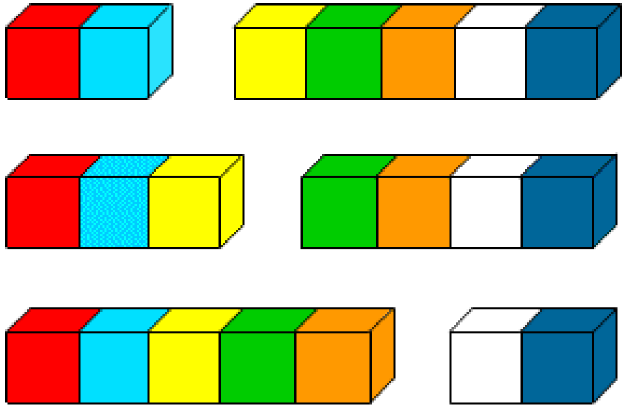
- https://nrich.maths.org/2284

You have a stick of 7 interlocking cubes. You cannot change the order of the cubes.



You break off a bit of it leaving it in two pieces.

Here are 3 of the ways in which you can do it:



In how many different ways can it be done?

Thank you for your participation

- Last evaluation to fill in!!