Aims of the 3 sessions:

- **Session 1**: Differences between Ratio and Proportion, links to fractions, decimals and percentages and identifying areas of multiplicative reasoning in the primary curriculum. Considering some models and images. Gap task

- **Session 2**: Singapore bar modelling and use of a table to support ratio and proportion problems. Equivalent ratios and fractions. Ideas for assessing understanding. Gap task planning a lesson.

- **Session 3**: Looking at scaling. Conversion graphs. Ratio and proportion in shape. Adapting recipes and developing reasoning further in R and P. What are the pitfalls? How can we support further?
Aims

- Reviewing the gap task
- Adapting recipes
- Investigating scale in shapes
- Looking at similar shapes
- Next steps
Gap task - Plan a lesson

- Think about a lesson or sequence of lessons with a specific learning objective connected to ratio and proportion

- Possibly looking at recipes, mixed problems, straight line graphs that show the connection between two things (particularly measures)

- Think about resources you might provide

- Design a concept cartoon or diagnostic question or other to assess pupils’ understanding
Answer 67
Question You exchange £60 for €72. How many pounds would you need to exchange to get €180?

Answer 120
Question A is inversely proportional to B. If B=60 when A=2, what is B when A = 10?

Answer 8
Question One cube had sides that are 3 times as long as another cube. How many times bigger is its volume?

Answer 27
Question The ratio 4:36 can be simplified to 1:9. What number would 1 be?

Answer 9
Question £500 is divided between Jonathan and Wanda in the ratio 3:7. How much in pounds does Wanda get?

Answer 2/3
Question 250 apples cost £72.50. How much in pounds does one cost?

Answer 0.29
Question A cheerio has a top speed of 220km/h. What would this be in m/s to the nearest whole number?

Answer 33
Question A box of marshmallows has 525g of its contents. If it now contains 340 marshmallows, how many would it have?

Answer 400
Question Red and blue paint is mixed in the ratio 0.3:1 to make purple. What fraction of the paint used was red?

Answer 2/5
Question £1 £850 is invested at an interest rate of 2% per year. How much would it be worth after 5 years to the nearest pound?
The ratio of girls to boys in a class is 1:3
What fraction of the class are girls?

A 1/3  B 1/4  C 3/4

A recipe for 6 people states 200g of butter
How much butter is needed for 9?

A 250g  B 400g  C 500g  D 300g

Bread rolls can be bought in different sized packs. Naz will buy whole packs that are all the same size. Which packs are the cheapest for 26 rolls?

A 12 rolls for £1.08
B 9 rolls for 90p
C 6 rolls for 54p
D 4 rolls for 38p
Concept cartoons

That means \( \frac{1}{5} \) like football

No! It's a \( \frac{1}{4} \)

1 in 5 children like football as their favourite sport.

That's the same as saying 1:5

mmm 1:4

What do you think?
Assessing understanding

- True, not true
- Diagnostic questions
- ASN
- Odd one out
- Concept cartoons
- Show me an example of... a 3:4 ratio, a proportion of dogs and cats that is \( \frac{3}{4} \)
- Hard and Easy
Is the Angel of the North Proportional?
The ‘Alternative’ Angel
Proportions of a human

- the whole foot from toe to heel is the same as the distance between the elbow and the wrist
- when a person kneels, they reduce their height by a quarter
- the length of a hand is one tenth of the person’s height
- the length of the ear is one third of the length of the face.

(Vitruvian Man Leonardo Da-Vinci)
Recipes and scales

Here are the ingredients for fish pie for two people.

Fish pie
(for 2 people)
250 g fish
400 g potato
25 g butter

Omar makes fish pie for 3 people. How many grams of fish should he use?

KS2 1997 Paper B level 5

Here are the ingredients for chocolate ice cream.

- cream 400 ml
- milk 500 ml
- egg yolks 4
- chocolate 120 g
- sugar 100 g

Stefan has only 300 ml of cream to make chocolate ice cream.

How much chocolate should he use?

Show your method

KS2 Sample paper for 2016 SATs
Research Paper 1

Didactical designs for students’ proportional reasoning: an “open approach” lesson and a “fundamental situation”

Takeshi Miyakawa & Carl Winsløw

Published online: 28 February 2009

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<td>1.666</td>
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![Diagram](image)
Research Paper 1 Continued

The paper focused on two lessons:

- A Japanese mathematics class
- A French mathematics class

Both classes were working on proportionality but took different approaches:

- An open approach
- A fundamental situation
Japanese lesson

The teacher showed 3 photographs, all in proportion and the children discussed what they noticed about the photographs.

They were then asked to:

* Draw a square with a dimension of 3cm
* Draw another square with a dimension of 5cm
* Are the two of the same form?
The Japanese Lesson Continued

Draw a rectangle of $3 \times 5$ cm and then one of size $5 \times 7$ cm

Are these of the same form?

Conjecturing and convincing then ensues.

Homework - “if you think the rectangles are of different form, construct another rectangle of the same form as the smaller one.”
T: So, Satoshi, what is your conclusion? They are of the same form?
S: Different form.
T: Different form? Why?
S: Because the ratios of the factors are different.

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The French Lesson

The situation was organized as follows:
• students were divided into groups of four to five
• each group was given a cardboard copy of the puzzle

Each member in the group had to enlarge at least one piece, or a pair may have enlarged two pieces. Then, the new (larger) pieces had to be assembled to a puzzle like the original one.
The French Lesson

Given a “puzzle” like the one shown, determine how to enlarge it such that the side of length 4 cm becomes 7 cm. Decide who is working on which piece.
Angles in a Triangle: 180°
Angles in a quadrilateral: 360°
All angles are 45°, 90° or 135°
What misconceptions might arise?

Note the lesson is planned to deliberately expose misconceptions.
What Happened?

• Students spontaneously tend to construct the larger pieces by adding 3 cm to all known sides (since 7 cm is 3 cm more than 4 cm).

• Other methods were then tried out.

The crucial point was that only the multiplication of all sides by $\frac{7}{4}$ will work in the sense that any other method would result in pieces which could not be assembled.
Activity

Can you sort the word problems into two groups: additive and proportional?

**Problem 1**
Peter and Tom are loading boxes in a truck. They load equally fast but Peter started later. When Peter has loaded 40 boxes, Tom has loaded 100 boxes. If Peter has loaded 60 boxes, how many boxes has Tom loaded?

**Problem 2**
Rachel and John are planting flowers. They started together but John plants faster. When Rachel has planted 4 flowers, John has planted 12 flowers. If Rachel has planted 20 flowers, how many flowers has John planted?

**Problem 3**
Ann and David are manufacturing dolls. They work equally fast but David started earlier. When Ann has made 12 dolls, David has made 24 dolls. If Ann has made 48 dolls, how many has David made?

**Problem 4**
Laura and Peter are planting stamps on postcards. They started together but Laura pastes slower. When Laura has pasted 80 stamps, Peter has pasted 280 stamps. If Laura has pasted 120 stamps, how many stamps has Peter pasted?
What strategies might you use to solve them?

- Share strategies
- What do these strategies have in common?
- How can we support our pupils?
Write the missing numbers.

60 months = [ ] years

72 hours = [ ] days

84 days = [ ] weeks
Tick two shapes that have \( \frac{3}{4} \) shaded.
11

Here is a rule for the time it takes to cook a chicken.

Cooking time = 20 minutes plus an extra 40 minutes for each kilogram

How many minutes will it take to cook a 3 kg chicken?

minutes

1 mark
Amina planted some seeds.

For every 3 seeds Amina planted, only 2 seeds grew.

Altogether, 12 seeds grew.

How many seeds did Amina plant?
A cat sleeps for **12 hours** each day.

50% of its life is spent asleep.

Write the missing percentage.

A koala sleeps for **18 hours** each day.

[ ] % of its life is spent asleep.
On a map, 1 cm represents 20 km.

The distance between two cities is **250 km**.

On the map, what is the distance between the two cities?
Here are two similar right-angled triangles.

Write the ratio of side $a$ to side $b$.

$a : b =$  

1 mark
3 pineapples cost the same as 2 mangoes.

One mango costs £1.35

How much does one pineapple cost?
Mixed problems

Making Comparisons:

1) Which class had the best attendance? The worst attendance?

<table>
<thead>
<tr>
<th>Class</th>
<th>Number in class</th>
<th>Number present</th>
<th>Percentage present</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>30</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>Class 2</td>
<td>25</td>
<td>23</td>
<td></td>
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<tr>
<td>Class 3</td>
<td>20</td>
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<td>Class 4</td>
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<td>Class 6</td>
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<td>Class 7</td>
<td>32</td>
<td>28</td>
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</tbody>
</table>

2) 6 tins of sweets cost £9. What will 8 tins cost?

3) 30 packets of crisps cost £1.40. What will 3 packets cost?

4) 8 metres of cloth cost £20. What will 5 metres cost?

5) 6 bottles of lemonade cost £7, what would 1 cost? £1.1666666 so £1.17

6) In measuring distance, we know that 5 miles is about 8 km. What other facts do we know? Make a graph to show this information and draw out three facts that can be shown from your graph.

7) In converting units of mass, we know that 1 Kg is 2.2 lb. Use this information to find other facts. Make a graph to show the relationship between Kg and pounds. By looking at your graph, what other facts can you now say?

8) A car can travel 120 km on 9 litres of petrol. How far will it travel on 21 litres of petrol?

9) If Steve drives at 72mph on average, how long will it take him to travel 48 miles to London and back?
Conversion graphs

Conversion graph: miles – km

Conversion graph: gallons – litres

Conversion graph: pounds – kilograms
You will need Worksheet 2134a.

**Similar Rectangles?**

1. Cut out rectangle A and rectangle B from the worksheet.

Place rectangle A on top of rectangle B.

Look at the diagonals.

Both sets of diagonals coincide.
Smile - STEM - Exploring similarity

For both rectangles A and B the ratio of the Long side : Short side is 2 : 1.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Long Side</th>
<th>Short side</th>
<th>Ratio Long side : Short side</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6</td>
<td>3</td>
<td>6 : 3 = 2 : 1</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>4</td>
<td>8 : 4 = 2 : 1</td>
</tr>
</tbody>
</table>

Rectangles are similar if the ratio Long side : Short side is the same.

Rectangle A is similar to rectangle B.

2. Cut out the other rectangles from the worksheet.
   Group them into 2 sets of similar rectangles.

3. Make a table for both sets to show the ratio of the Long side : Short side.
   Check that they are similar rectangles.

4. For each set draw another rectangle that is similar.
   Check that they are similar rectangles.
Smile - STEM - Exploring similarity

Cut out these rectangles.

![Smile Worksheet 2134a](image)

- A
- B
- C
- D
- E
- F
- G
- H
Proportionality and shape

Always, Sometimes, Never:

“All rectangles are similar”
Which of these rectangles are similar?

How could you convince someone?
Using Proportionality to find unknown sides in similar triangles

Big triangle       Small triangle
20                16
20 \times \frac{16}{20} \quad \text{is} \quad 16
a \times \frac{4}{5} \quad 12 \times \frac{5}{4} = 15

Find the length of side a
What method might you use to solve this problem?
Using proportionality to find unknown sides
Similar Triangles

What is the relationship between these triangles?
Similar Triangles

What is the relationship between these triangles?
Similar Triangles

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What is the relationship between these triangles?
Similar Triangles

What is the relationship between these triangles?

Does this proportionality only apply to right angled triangles?
Pentominoes
Pentomino Blowups

- It is difficult, but possible to cover the figure with pentominoes. Try to do it.
- How many pentominoes did it take to cover it?
- By doubling the dimensions of the other pentominoes, you can create doubled pentomino puzzles. Altogether, ten of the twelve doubled pentomino puzzles can be solved with pentominoes from one set.
- All tripled pentomino puzzles can be solved with pentominoes from one set. (In fact all can be solved without using the pentomino that is being tripled.)
- How many pentominoes does it take to cover a tripled pentomino puzzle?
More pentominoes
Next steps for own CPD

- What will you think about further to these sessions?
- What resources may you want to develop?